CAE

Math Sample Items

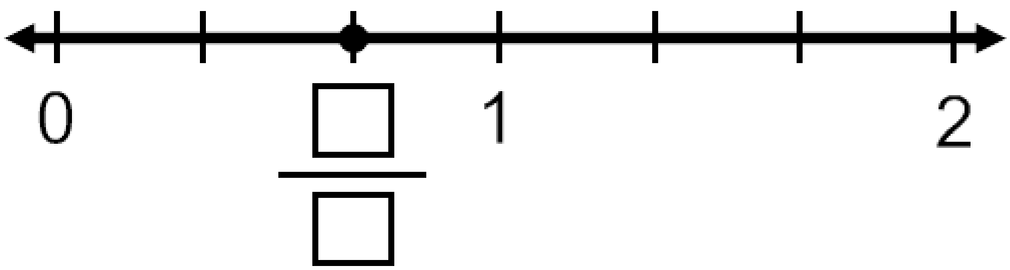
3rd Grade – Technology-Enhanced (Fill-in-the-Blank)

CCSS.MATH.CONTENT.3.NF.A.2
Understand a fraction as a number on the number line; represent fractions on a number line diagram.

> CCSS.MATH.CONTENT.3.NF.A.2.B
> Represent a fraction *a*/*b* on a number line diagram by marking off a lengths 1/*b* from 0. Recognize that the resulting interval has size *a*/*b* and that its endpoint locates the number *a*/*b* on the number line.

What fraction represents the value of the point that is plotted on the number line shown?

Type the numbers in the boxes.



Key: $\frac{\boxed{2}}{\boxed{3}}$ or equivalent

The student understands that the tick marks are spaced so that the distance between tick marks is $\frac{1}{3}$. Since the point is plotted 2 tick marks away from 0, the point represents $\frac{2}{3}$.

5th Grade – Selected Response

CCSS.MATH.CONTENT.5.NF.B.6
Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

Ravi is cooking soup for a group of people. The recipe calls for $2\frac{1}{4}$ cups of tomato sauce for one batch of soup. If Ravi estimates that he needs to cook $3\frac{1}{2}$ batches of soup, how many cups of tomato sauce should he use?

A. $5\frac{2}{6}$

B. $5\frac{3}{4}$

C. $6\frac{1}{8}$

D. $7\frac{7}{8}$

Rationales

A. The student added the mixed numbers instead of multiplying them. Additionally, the student incorrectly added the mixed numbers by adding the whole numbers together, and then adding the numerators together and the denominators together.

$$2\frac{1}{4} + 3\frac{1}{2} = 5\frac{2}{6}$$

B. The student added the mixed numbers instead of multiplying them.

$$2\frac{1}{4} + 3\frac{1}{2} = 5 + \frac{1}{4} + \frac{1}{2} = 5 + \frac{1}{4} + \frac{2}{4} = 5 + \frac{3}{4} = 5\frac{3}{4}$$

C. The student incorrectly multiplied the mixed numbers together. The student first multiplied the whole numbers and then multiplied the fractions.

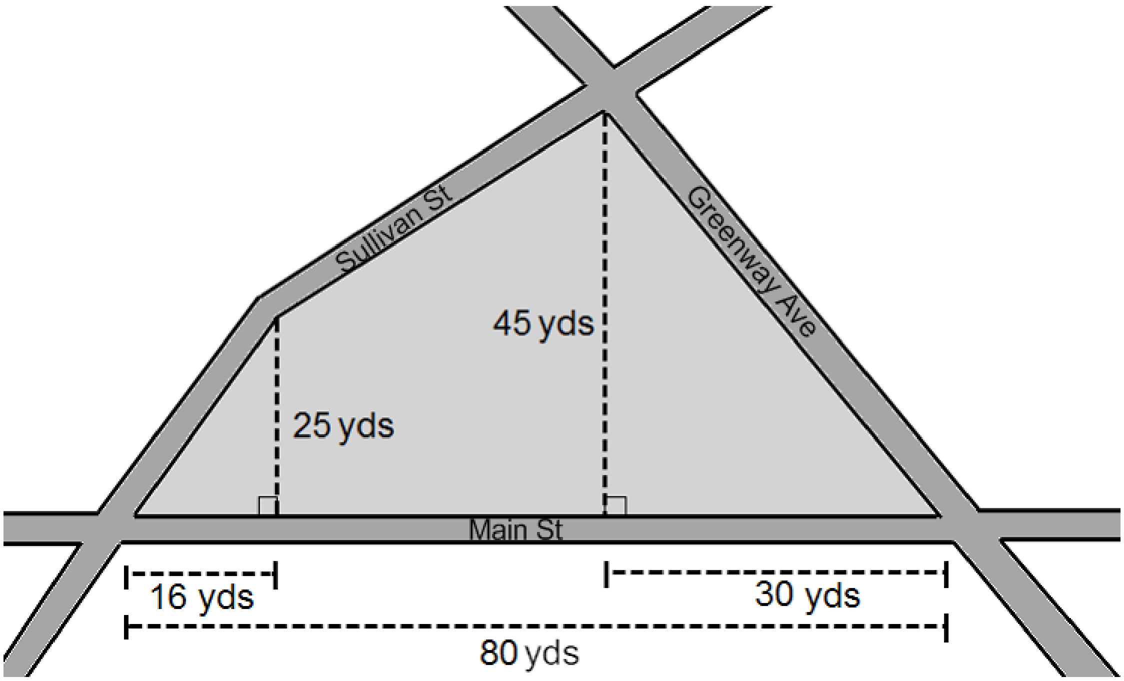
$$2\frac{1}{4} \bullet 3\frac{1}{2} = (2 \bullet 3) + \left(\frac{1}{4} \bullet \frac{1}{2}\right) = 6 + \frac{1}{8} = 6\frac{1}{8}$$

D. Correct. The student correctly determined that since each batch of soup requires $2\frac{1}{4}$ cups of tomato sauce, $3\frac{1}{2}$ batches of soup would require $2\frac{1}{4} \bullet 3\frac{1}{2}$ cups of tomato sauce.

$$2\frac{1}{4} \bullet 3\frac{1}{2} = \frac{9}{4} \bullet \frac{7}{2} = \frac{63}{8} = 7\frac{7}{8}$$

CCSS.MATH.CONTENT.6.G.A.1
Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

A quadrilateral-shaped parking lot is bordered by three roads, as shown in the figure.



What is the area, in square yards, of the parking lot?

A. 2,065

B. 2,225

C. 2,405

D. 2,940

Rationales

A. Correct. The student dissected the quadrilateral into two right triangles and one trapezoid. The student found the area of each shape and then added them together.

$$\frac{16 \cdot 25}{2} + \frac{30 \cdot 45}{2} + \frac{(45 + 25)}{2} \cdot 34$$

$$= 200 + 675 + 1190 = 2{,}065 \text{ sq yds}$$

B. The student understood that the area of the quadrilateral is equal to the sum of the areas of the two right triangles and one trapezoid. The student correctly found the area of each triangle but used the formula for the area of a rectangle instead of the formula for the area of a trapezoid.

$$\frac{16 \cdot 25}{2} + \frac{30 \cdot 45}{2} + 30 \cdot 45$$
$$= 200 + 675 + 1350 = 2{,}225 \text{ sq yds}$$

C. The student may have understood that the area of the quadrilateral is equal to the sum of the areas of the two right triangles and one trapezoid. The student correctly found the areas of the two right triangles but incorrectly applied the formula for the area of a rectangle to the area of a trapezoid.

$$\frac{16 \cdot 25}{2} + \frac{30 \cdot 45}{2} + (45 \cdot 34)$$
$$= 200 + 675 + 1530 = 2{,}405 \text{ sq yds}$$

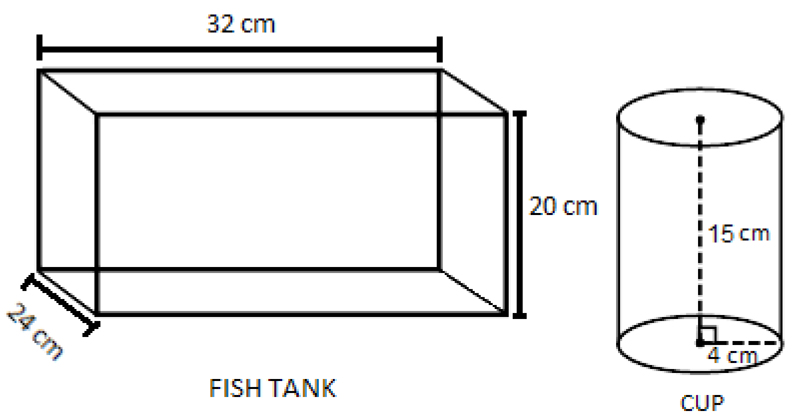
D. The student may have understood that the area of the quadrilateral is equal to the sum of the areas of the two right triangles and one trapezoid. The student correctly found the area of the trapezoid but did not divide by 2 when finding the area of each triangle.

$$\frac{(45 + 25)}{2} \cdot 34 + (16 \cdot 25) + (45 \cdot 30)$$
$$= 1190 + 400 + 1350 = 2{,}940 \text{ sq yds}$$

CCSS.MATH.CONTENT.7.G.B.6
Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

CCSS.MATH.CONTENT.8.G.C.9
Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

Andy has a small fish tank in the shape of a right rectangular prism, as shown in the figure. He uses the cylindrical cup shown to fill the fish tank with water. What is the minimum number of times Andy will need to put water into the cup and pour it into the fish tank in order to completely fill the fish tank? Use 3.14 for π. (1 cubic centimeter (cm^3) of water is equivalent to 1 milliliter (mL).)



The minimum number of times that Andy has to put water into the cup and pour it into the fish tank is the number of completely filled cups of water that will total the volume of the fish tank.

$$V_{\text{fish tank}} = 32 \cdot 24 \cdot 20 = 15360 \text{ cm}^3 \text{ or } 15{,}360 \text{ mL}$$

$$V_{\text{cup}} = \pi \cdot 4^2 \cdot 15 = 240\pi \text{ in}^3 \text{ or } 240\pi \text{ mL}$$

$$\frac{15360 \text{ mL}}{240 \cdot 3.14 \text{ mL}} \approx 20.38 \text{ times}$$

Andy will have to completely fill and empty the cup at least 20 times, and then partially fill and empty the cup one additional time in order to completely fill the empty fish tank with water. Therefore, the minimum number of times Andy has to put water into the cup and pour it into the fish tank 21 times.

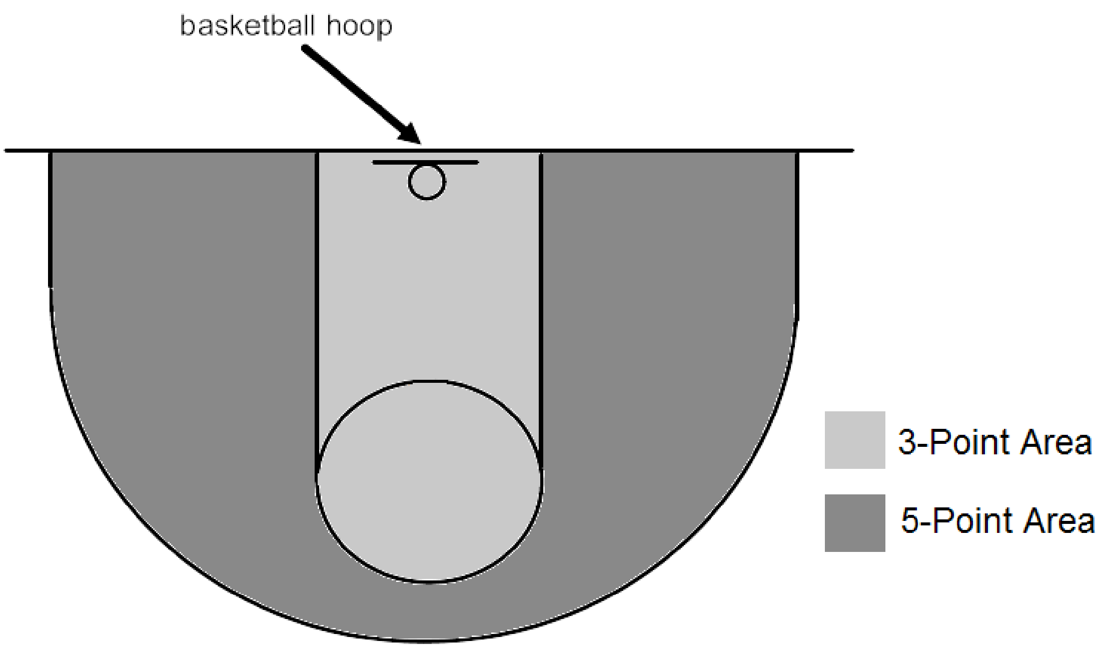
Algebra I – Constructed Response

CCSS.MATH.CONTENT.HSA.REI.C.6
Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

CCSS.MATH.CONTENT.HSA.CED.A.2
Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

CCSS.MATH.CONTENT.HSA.CED.A.3
Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.

Jorge and Rina are playing a variation of a basketball game. In their game, each basket that is made is worth either 3 points or 5 points depending on the area from which Jorge or Rina shoot the basketball. The diagram shows the 3-point and 5-point areas on the basketball court.



Jorge claims that he made a total of 26 baskets and has a total of 83 points, but Rina disagrees. Create and solve a system of equations to prove whether Jorge's claim is possible or not.

Exemplar:
Let x represent the number of 3-point baskets that Jorge made.
Let y represent the number of 5-point baskets that Jorge made.

$$3x + 5y = 83$$
$$x + y = 26$$

$$\begin{array}{r} 3x + 5y = 83 \\ -3(x + y = 26) \\ \hline 2y = 5 \\ y = 2.5 \end{array}$$

Jorge's claim is incorrect because this would mean that he made 2-and-a-half 5-point baskets. This is not possible because the number of baskets made must be a whole number.

Algebra I – Technology-Enhanced (Fill-in-the-Blank)

CCSS.MATH.CONTENT.HSF.IF.A.2
Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

CCSS.MATH.CONTENT.HSF.IF.C.8.A
Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

A ball is launched into the air where its height above the ground can be represented by the function $h(t) = -16t^2 + 64t + 6$, where t is the number of seconds since the ball had been launched.

Complete the function to show the number of seconds at which the ball reaches its maximum height above the ground.

Type the correct numbers in the boxes.

$h(\square) = \square$

Key: $h(\boxed{2}) = \boxed{70}$

The student was able to correctly determine the maximum height of the function and the number of seconds at which that maximum occurs. The student may have done this by completing the square and rewriting the function in vertex form.

$$
\begin{aligned}
h(t) &= -16t^2 + 64t + 6 \\
&= -16\left(t^2 - 4t\right) + 6 \\
&= -16\left(t^2 - 4t + 4\right) + 6 + 64 \\
h(t) &= -16(t - 2)^2 + 70
\end{aligned}
$$

An alternative way to find the maximum would be to find the x-value of the vertex by computing $t = \frac{-b}{2a}$ and then solving for $h(t)$ at that t-value.

$$
t = \frac{-b}{2a} = \frac{-64}{2(-16)} = \frac{-64}{-32} = 2
$$

$$
\begin{aligned}
h(2) &= -16(2)^2 + 64(2) + 6 \\
&= -64 + 128 + 6 \\
&= 70
\end{aligned}
$$

The student understands that 2 seconds after the ball has been launched, it reaches its maximum height of 70 feet. The student understands how to write this in function notation, where the t value represents the number of seconds at which the ball reaches its maximum height of $h(t)$ feet.

CCSS.MATH.CONTENT.7.EE.A.2
Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. For example, $a + 0.05a = 1.05a$ means that "increase by 5%" is the same as "multiply by 1.05."

CCSS.MATH.CONTENT.HSA.SSE.A.1.A
Interpret parts of an expression, such as terms, factors, and coefficients.

Jane currently has $1,000 in a savings account that accrues interest annually. She does not use the account to withdraw or deposit money. The amount of money in her savings account after *t* years can be modeled by the expression $1000(1.001)^t$. What is the annual interest rate of Jane's savings account?

A. 0.001%

B. 0.01001%

C. 0.1%

D. 1.001%

Rationales

A. The student chose the value after the decimal point in the term "1.001." However, 0.001 does not reflect the percentage of the money in the account that is being added to the account every year.

B. The student may have understood that the decimal point in the term "1.001" needed to be moved two places so that it would be converted to a percentage. However, the student moved the decimal point two places in the wrong direction. The student may also not have understood what the 1 before the decimal represents.

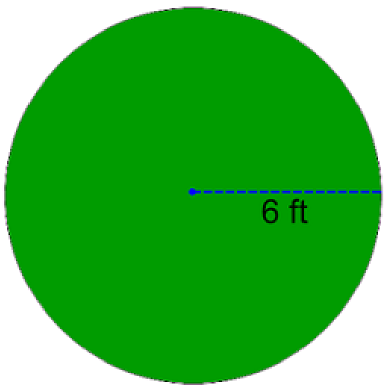
C. Correct. The student understood that each year, 0.001 times the amount in the account is added into the account as interest. This, however, must be converted to a percentage by moving the decimal point two places to the right. Therefore, 0.1% of the money in the account is added to the account every year.

D. The student chose the number "1.001," which is shown in the expression. The student may not have an understanding of what this number represents.

CCSS.MATH.CONTENT.HSN.Q.A.1
Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

CCSS.MATH.CONTENT.HSG.MG.A.3
Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).*

A rotating sprinkler is placed in a garden so that it automatically waters a circular area of grass. The sprinkler has a reach of 6 feet.



It takes the sprinkler 12 seconds to make one complete rotation. What is the rate at which the sprinkler waters the grass?

A. π sq ft per second

B. 3π sq ft per second

C. 12π sq ft per second

D. 36π sq ft per second

Rationales

A. The student found the unit rate at which the sprinkler waters the perimeter, not the area.

$$P = 2\pi r = 2\pi \cdot 6 = 12\pi$$

$$\frac{12\pi \text{ ft}}{12 \text{ sec}} = \frac{x}{1 \text{ sec}}$$

$$x = \frac{\pi \text{ ft}}{1 \text{ sec}}$$

B. Correct. The student correctly found the area of the circle. Then, the student correctly calculated the unit rate at which the sprinkler waters the area of grass.

$$A = \pi r^2 = \pi(6)^2 = 36\pi \text{ ft}^2$$

$$\frac{36\pi \text{ ft}^2}{12 \text{ sec}} = \frac{x}{1 \text{ sec}}$$

$$x = \frac{3\pi \text{ ft}^2}{1 \text{ sec}}$$

C. The student may have chosen the value that represents the perimeter of the circle.

$$P = 2\pi r = 12\pi \text{ ft}$$

Or, the student may have incorrectly used the formula for the area of a circle and then computed the unit rate at which the sprinkler waters that incorrect area.

$$A = \pi d^2 = \pi(12)^2 = 144\pi \text{ ft}^2$$

$$\frac{144\pi \text{ ft}^2}{12 \text{ sec}} = \frac{x}{1 \text{ sec}}$$

$$x = \frac{12\pi \text{ ft}^2}{1 \text{ sec}}$$

D. The student correctly found the area of the circle but did not calculate the unit rate at which the sprinkler waters the area of grass.

$$A = \pi r^2 = \pi(6)^2 = 36\pi \text{ ft}^2$$

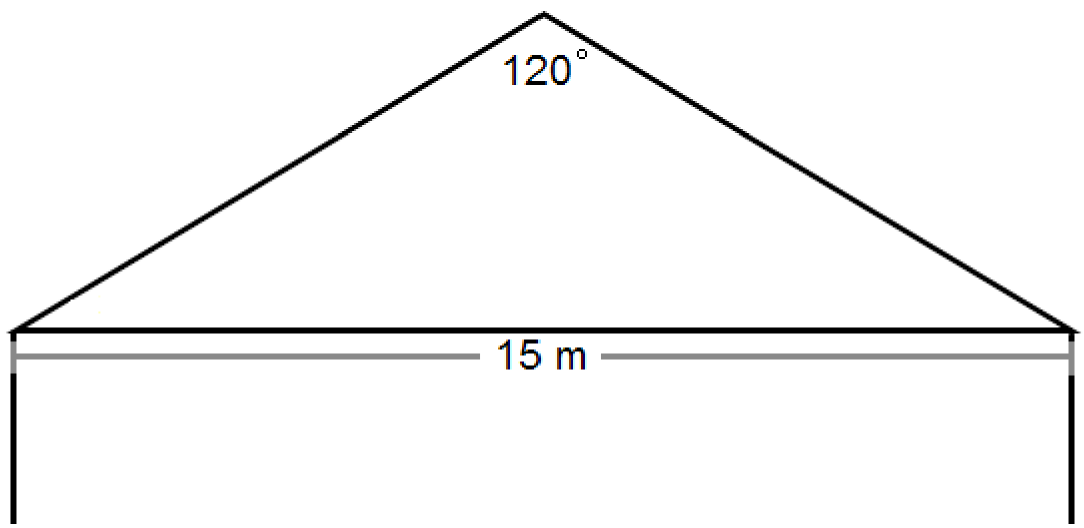
High School Geometry – Modeling Constructed Response

CCSS.MATH.CONTENT.HSG.SRT.C.8
Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.*

CCSS.MATH.CONTENT.HSG.MG.A.3
Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).*

CCSS.MATH.CONTENT.HSG.SRT.B.5
Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

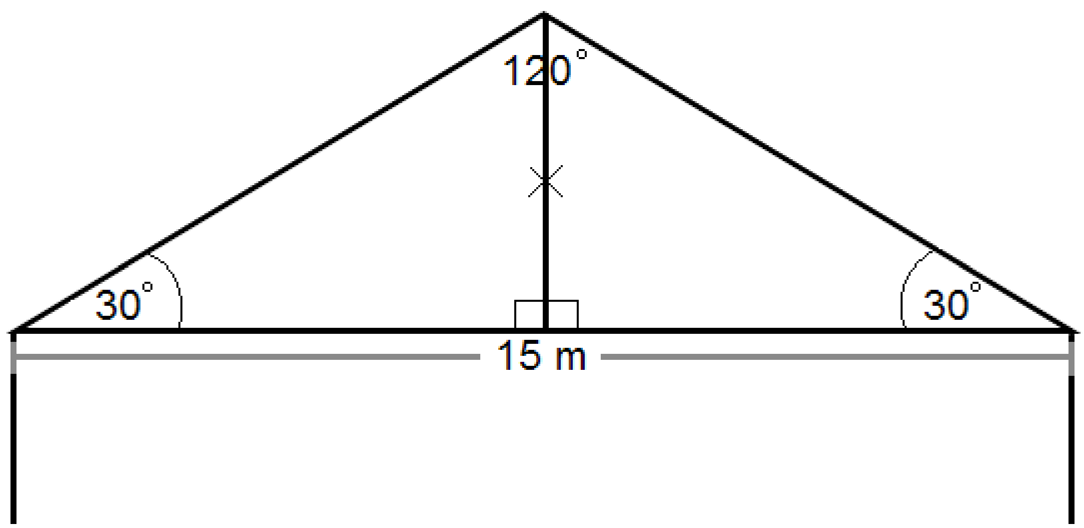
An architect is designing a house that will have a roof in the shape of an isosceles triangular prism. The front of the house is shown in the figure. The architect plans for the roof to make a 120° angle at the very top of the house. If the length across the front of the house is 15 meters, what will be the height of the roof that the architect is designing? Round your answer to the nearest hundredth of a meter. Show your work and explain your reasoning.



Exemplar

The bottom two angles of the triangle shown are congruent since it is given that each base of the triangular prism roof is an isosceles triangle. Since the sum of the interior angles of a triangle is 180°, the base angles of the isosceles triangle must be 30° each.

Drawing a line from the top of the triangle to its base will show the altitude, or height, of the roof. This altitude is perpendicular to the base of the triangle.



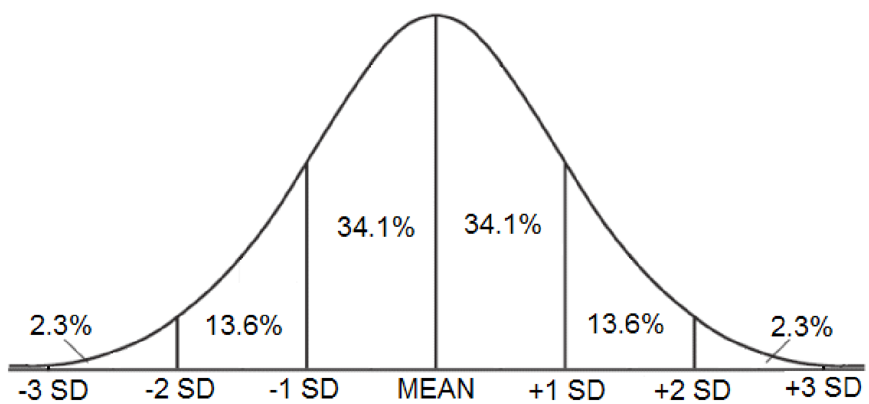
The altitude is congruent to itself by the reflexive property. Since the altitude is perpendicular to the base, it forms two right angles that are congruent. And it has already been established that the base angles of the isosceles triangle are each 30°. Therefore, the altitude forms two congruent right triangles by the AAS postulate.

Since corresponding parts of congruent triangles are congruent, the base of each right triangle is 7.5 meters. Given the bottom length of the right triangle, a trigonometric ratio can be created to determine the height of the roof.

$$\tan 30° = \frac{x}{7.5}$$
$$x = 7.5 \tan 30° \approx 4.33 \text{ m}$$

CCSS.MATH.CONTENT.HSS.ID.A.4
Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

Mr. Farrell gave his classes a final exam where students could score anywhere from 0 to 200. The students' scores approximated a normal distribution with a mean score of 168 and a standard deviation of 6 points.



Complete the following three sentences.

Drag and drop the correct percentage to each box.

- The probability that a randomly selected student scored over 174 is approximately ☐.
- The probability that a randomly selected student scored less than 156 is approximately ☐.
- The probability that a randomly selected student scored between 168 and 180 is approximately ☐.

2.3%
13.6%
15.9%
34.1%
47.7%
68.2%
84.1%
97.7%

Answer

- The probability that a randomly selected student scored over 174 is approximately $\boxed{15.9\%}$.
- The probability that a randomly selected student scored less than 156 is approximately $\boxed{2.3\%}$.
- The probability that a randomly selected student scored between 168 and 180 is approximately $\boxed{47.7\%}$.